System Reliability in Health Care Systems

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Abstract: Health Information enhances the quality and efficiency of Health Care Systems. The findings provide an important base line for future evaluation. In this paper, we discuss System Reliability in Health Care Systems. The knowledge of System Reliability concepts helps the Systems in Health Care to be more perfect and improve its status for the betterment of the Society. Here we make use of the System Reliability of Mixed Parallel Configuration Systems and using Fault-tree Analysis approach determine the Reliability of Child-birth Status of Pregnant Women affected by Anemia and Reliability of cure of Typhoid disease affected persons in a particular area. This type of studies enables us to take preventive measures in Health Care Systems.

Key Words: System Reliability, Reliability Importance, Parallel Configuration Systems, Fault-tree Analysis Approach

1. Introduction

System Reliability Approach to problems in Medical field helps us to analyse the overall Reliability of Systems in Healthcare, from the reliability factors of subsystems and elements [2]. System Reliability is defined as the Probability that a System will perform its intended function during a specified period of time under stated conditions. One of the purposes of System Reliability Analysis is to identify the weakness in a System and to quantify the impact of component failure [4]. The so-called Reliability Improvement is used for this purpose. Institute for Health Improvement has taken the initiative to apply industry methods of System Reliability to Health Care Systems. Reliability Importance measures provide a numerical rank to determine which Components are more important to System Reliability Improvement and also to identify the components that are more critical to System Failure. In order to evaluate the importance of different aspects for a System, a set of importance measurements such as Birnbaum Component Importance, Reliability Criticality Importance, Upgrading Function and Operational Criticality Importance [3, 5] have been well defined and widely used in Engineering Practice.

Generally, Reliability Importance is a function of operation time, of Failure and Repair Characteristics (of all Components in the System) and of the System structure. So far, all Reliability Importance indices are calculated through combinatorial approaches such as Reliability Block Diagram or Fault Tree Analysis (FTA), or structure function or Markov Modelling [4].

In general, Importance ranks the Components in a System using a numerical rank (relative importance), based on certain System characteristic of interest, such as the Component's contribution to a System (failure) event occurrence. Birnbaum first introduced the concept of Importance in 1969, and one of the most widely used Reliability Importance indices is Birnbaum's Component Importance.

In this paper, we make use of the System Reliability of Mixed Parallel Configuration System using Fault-Tree Analysis approach for determining the Reliability of Child-birth Status of Pregnant Women affected by Anemia and Reliability of cure of Typhoid disease affected persons in a particular area. Such kind of studies enables us to take preventive measures in Health Care Systems.

Anemia is one of the most commonly encountered medical disorders during pregnancy. In developing countries it is a cause of serious concern as, besides many other adverse effects on the mother and the fetus. It contributes significantly high maternal mortality. According to World Health Organization estimates, up to 56% of all women living in developing countries are anemic. In India, National Family Health Survey -2 in 1998 to 99

shows that 54% of women in rural and 46% women in urban areas are anemics. Iron deficiency anemia (IDA) is the commonest type of anemia in pregnancy. As most women start their pregnancy with anemia or low iron stores, so prevention should start even before pregnancy. The Ministry of Health, Government of India has now recommended intake of 100 mg of elemental iron with 500 mg of folic acid in the second half of pregnancy for a period of at least 100 days. Women who receive daily antenatal iron supplementation are less likely to have iron deficiency anemia at term.[8]

There are three types of Anemia, Mild, Moderate and Severe Anemia. Women with chronic mild anemia may go through pregnancy and labor without any adverse consequences, because they are well compensated. Women with moderate anaemia have substantial reduction in work capacity and may find it difficult to cope with household chores and child care. Available data from India and elsewhere indicate that maternal morbidity rates are higher in women with Hb below 8gm/dl. Premature births are more common in women with moderate anaemia. They deliver infants with lower birth weight and perinatal mortality is higher in these babies.. Anaemia directly causes 20 percent of maternal deaths in India and indirectly accounts for another 20 per cent of maternal deaths.

Another important disease is Typhoid fever which is very difficult to diagnose. Untreated Typhoid fever has a high mortality. It occurs mainly in rural and urban squatter settlements. Some People may require hospitalization and treatment with antibiotics. Others who may not show symptoms of typhoid but are carriers of the disease will also require treatment with antibiotics. Drugs such as Ciprofloxacin, Oflaxacin, Cefixime are being prescribed to cure this disease[7, 9].

The paper is organized as follows. Section 2 gives the basic definitions and related concepts. Section 3 deals with Mixed Parallel Configuration System and its Improvement by using Fault-tree Analysis Approach. In Section 4, we apply Mixed Parallel Configuration System using Fault Tree Analysis Approach to find out the Reliability of the Health Status of Pregnant Women and Reliability of cure of Typhoid Fever in a particular area with Numerical Examples. Section 5 draws the conclusion.

2. Basic Definitions

2.1 Reliability

Reliability is defined as the probability of a device (or an item) performing its purpose for the period intended under the given operating conditions. It can also be defined as the probability of non-failure. If F(t) is the failure probability, then $1-F(t)$ gives the non-failure probability. Thus, the Reliability of the device (or an item) for time T = t (ie., the device functions satisfactorily for time $T \geq t$ is

$$R(t) = 1 - F(t)$$
$$= 1 - \int_{-\infty}^{t} f(x)\,dx$$

Or

$$R(t) = \int_{t}^{\infty} f(x)\,dx$$

where $R(t)$ is the Reliability at time t

2.2 Fault Tree Analysis (FTA) approach

FTA is a top-down approach of a system analysis that is used to determine the possible occurrence of undesirable events or failures. Over the years, the method has gained favour over other reliability analysis approaches because of its versatility in degree of detail of complex systems. There are many symbols used to construct fault trees. The basic four symbols are

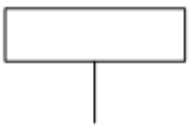


Fig. 1

Fig. 1 denotes a fault event that occurs from the logical combination of fault events through the input of logic gates such as OR and AND

Fig. 2 denotes a basic fault event

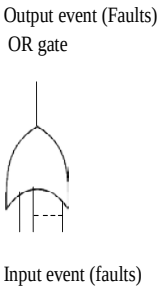


Fig. 3 denotes the output fault event if one or more of input fault events occur.

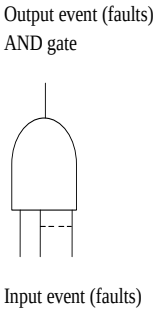


Fig 4

Fig. 4 denotes that an output fault tree event occurs if all the input fault events occur

2.3 Reliability Importance

One of the most widely used Reliability Importance indices is Birnbaum's Component importance[5]. Analytically this is defined by

$$I_k^B(t) = \frac{\partial R_S(t)}{\partial R_k(t)} = \frac{\partial F(t)}{\partial F_k(t)}$$

where

I_k^B is Reliability Improtance of the k^{th} component, $R_S(t)$ and $F(t)$ are the System Reliability and Unreliability at time t, respectively.

3. Mixed Configuration Models

3.1PARALLEL-PARALLEL CONFIGURATION (PP Model)

Consider a System which has two Sub Systems X_i (i = 1, 2...) connected in Parallel. Each Subsystem has three sub units connected in Parallel ($X_{i\,j}$, i = 1,2 and j = 1, 2, 3) All the units involved in this System are independent and the base events probabilities $P(X_{i\,j})$ are determined.

Fault-tree Analysis diagram for a Parallel – Parallel Configuration System (PP) Model is given below.

3.2 Fault-Tree Analysis diagram

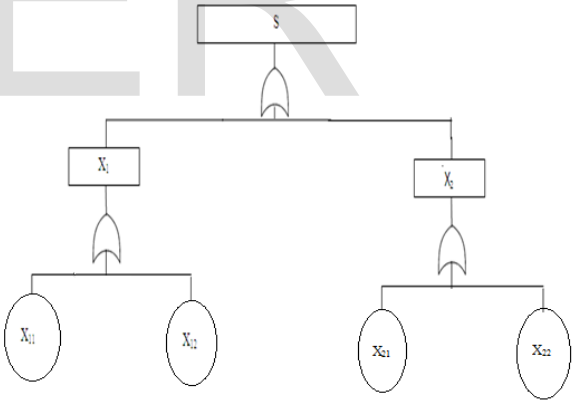


Fig. 5

The Reliability for Parallel-Parallel Configuration is given by

$$R(s) = 1 - \prod_{i=1}^{2}\left[1 - \left[1 - \prod_{j=1}^{2}(1 - P_{ij})\right]\right] \quad (1)$$

Substituting $P(X_{ij}) = r_{ij}$ and expanding the formula

$$R(S) = 1 - \left[\left[1-\left[r_{11}(1-r_{12})+r_{12}(1-r_{11})+r_{11}r_{12}\right]\right]\left[1-\left[r_{21}(1-r_{22})+r_{22}(1-r_{21})+r_{21}r_{22}\right]\right]\right] \quad (2)$$

The above formula can be given as

$$R(s) = 1 - \prod_{i=1}^{2}\left[1-\left[\sum_{j=1}^{2} r_{ij}(1-r_{ij+1}) + \prod_{j=1}^{2} r_{ij}\right]\right] \quad (3)$$

where j+1=1 for j= 2. Hence $r_{13} = r_{11}$, $r_{23} = r_{21}$.

In general for Parallel-Parallel Configuration System with m Sub Systems (i=1,2,…m) and n Sub units (j = 1,2,…n)

$$R(s) = 1 - \prod_{i=1}^{m}\left[1-\left[\sum_{j=1}^{n} r_{ij}(1-r_{ij+1}) + \prod_{j=1}^{n} r_{ij}\right]\right] \quad (4)$$

where j + 1 = 1, for j = n, Hence $r_{ij+1} = r_{i1}$

The Reliability Importance of Independent Components in the Systems are derived

$$\frac{\partial R}{\partial r_{1k}} = \left|\left(1-\sum_{\substack{j=1\\ j\neq k}}^{n} r_{1j} + \prod_{\substack{j=1\\ j\neq k}}^{n} r_{1j}\right)\left(\prod_{i=1}^{m}\left[1-\left(\sum_{j=1}^{n} r_{ij}(1-r_{ij+1})\right)+\prod_{j=1}^{n} r_{ij}\right]\right)\right| \quad (5)$$

$$\frac{\partial R}{\partial r_{2k}} = \left|\left(1-\sum_{\substack{j=1\\ j\neq k}}^{n} r_{2j} + \prod_{\substack{j=1\\ j\neq k}}^{n} r_{2j}\right)\left(\prod_{i\neq 2}^{m}\left[1-\left(\sum_{j=1}^{n} r_{ij}(1-r_{ij+1})\right)+\prod_{j=1}^{n} r_{ij}\right]\right)\right| \quad (6)$$

In general

$$\frac{\partial R}{\partial r_{ik}} = \left|\left(1-\sum_{\substack{j=1\\ j\neq k}}^{n} r_{ij} + \prod_{\substack{j=1\\ j\neq k}}^{n} r_{ij}\right)\left(\prod_{i\neq 1}^{m}\left[1-\left(\sum_{j=1}^{n} r_{ij}(1-r_{ij+1})\right)+\prod_{j=1}^{n} r_{ij}\right]\right)\right| \quad (7)$$

3.3 Numerical Example

Consider an example for a Parallel- Parallel Configuration System related to Medical Field which deals with the Reliability of Pregnant women in a particular area who are affected by Anemia. This study helps to take Connective measures to improve the efficiency of Health Condition of Anemia affected women.

Pregnant women whose Anemic is below level, are prescribed by the doctors either Zincofer or Orofer according to their health condition likewise, when the anemic is above level they are prescribed either ferrum plus monthly twice or folic tablets daily till delivery. The Fault Tree Analysis diagram of this Medical problem is given below:

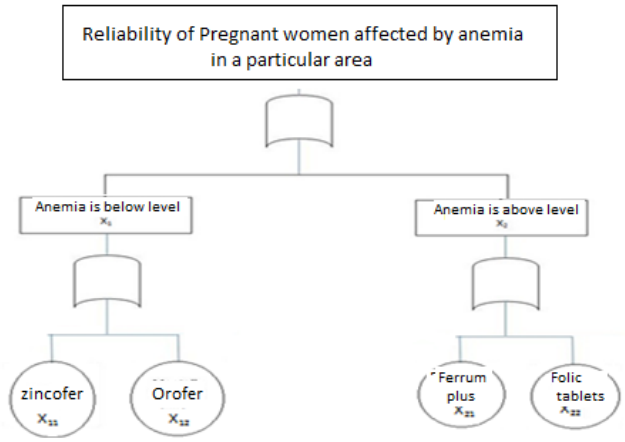


Fig. 6

Let us assume the Probability of the base events for a small sample as,

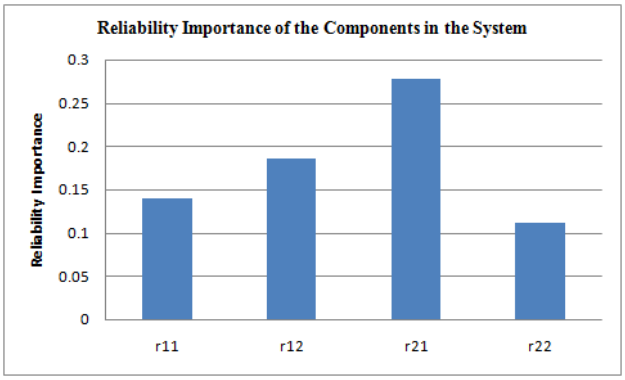
$r_{11} = 0.333$ $r_{12} = 0.5$

$r_{21} = 0.666$ $r_{22} = 0.1666$

Using (3) the Reliability of Pregnant women in a particular area is determined as R(s) = 0.9072

$$\frac{\partial R}{\partial r_{11}} = 0.1392 \qquad \frac{\partial R}{\partial r_{12}} = 0.1857$$

$$\frac{\partial R}{\partial r_{21}} = 0.2779 \qquad \frac{\partial R}{\partial r_{22}} = 0.1114$$



From the graph it is clear that the Reliability Importance of Orofer and Ferrum Plus are very high and these Components' Reliability can be improved in order to increase the Reliability of Pregnant women affected by anemia in a particular area.

Example 2

Consider the Parallel- Parallel Model of a Medical System which deals with the Reliability of the cure of Typhoid Disease affected persons in an area who are either in the initial or advanced stage. Initial stage can be treated with either ciprofloxacin or oflaxacin for period of 3-5 days and advanced stage can be treated with cefixime + oflaxacin or cefpodoxime + oflaxacin for a period of 7 days. The Fault Tree Analysis diagram is given below.

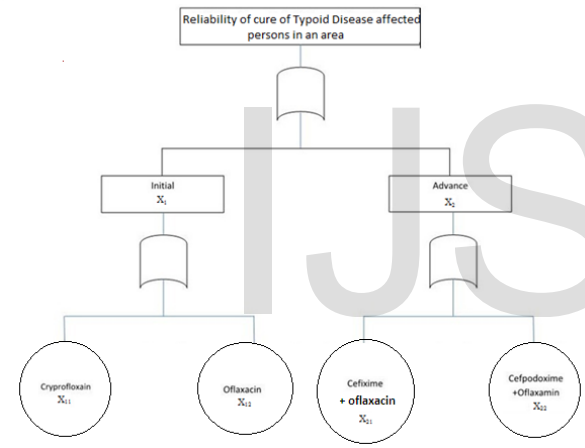


Fig. 7

Let us assume the Reliabilities of the base events for a small sample as

$r_{11} = 0.3$ $r_{12} = 0.7$

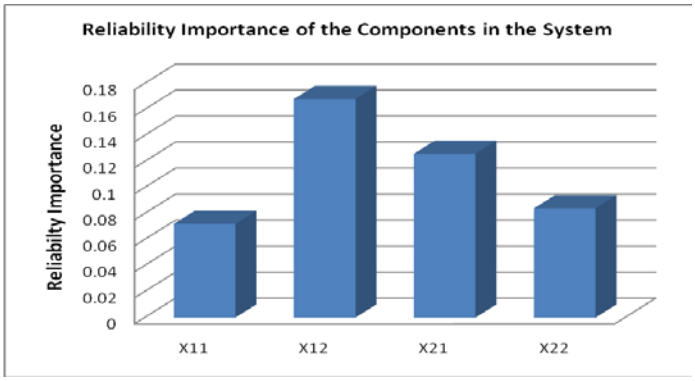
$r_{21} = 0.6$ $r_{22} = 0.4$

Using (3) the Reliability of the cure of Typhoid Disease affected person in an area is determined as

R(s) = 0.9496

$$\frac{\partial R}{\partial r_{11}} = 0.072 \qquad \frac{\partial R}{\partial r_{12}} = 0.168$$

$$\frac{\partial R}{\partial r_{21}} = 0.126 \qquad \frac{\partial R}{\partial r_{22}} = 0.084$$



From the graph, it is clear that the Reliability Importance of the Oflaxacin and Cefixime + Oflaxacin are very high, and these Components' Reliability can be improved in order to increase the Reliability of the cure of Typhoid Disease affected persons in an area.

4. Conclusion

In this paper, we have derived the System Reliability and Reliability Importance of the Components in the System of Mixed Parallel Configuration. We analyzed this with Numerical Examples from the Medical field. We find that the Reliability Importance of few Components in the System is very low and those Components' Reliability has to be improved for the efficiency of the System.

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6.Biographies

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